

Possible Worlds in "The Craft of Formal Logic"

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This paper investigates Prior's first original ideas in modal logic and the early formulation of the concept of possible worlds in his unpublished manuscript "The Craft of Formal Logic". "The Craft" was written during 1950 and 1951, when Prior was becoming familiar with the new approaches to old logic themes and the emerging formal techniques. Intended as a modern logic textbook (Geach and Kenny, 1976), the manuscript included a separate part devoted to modal logic, which represents the first text in modal logic he wrote (Copeland 1996). It contains a selection of topics that are canvassed from a historical perspective: decomposition of modal sentences; modal operators and negation; ambiguity of the "possible" and various types of necessity; similarity in the behaviour between modal operators and quantifiers; different interpretations and possible applications of the operators; properties and ordinary attribution; etc. The presentation of each topic is interwoven with his own comments, many of which are his initial and spontaneous responses and solutions to problems in modal logic that will preoccupy his later years. We see among them how the idea of possible worlds was conceived. Although Prior talks about these "entities" only informally, and hesitates how to name them, he is nevertheless very clear and explicit that they should become a fundamental concept of modal logic. Besides possible worlds, on the pages of "The Craft" we find anticipations of other significant developments in modal logic, particularly in the area of its application to the analysis of natural languages. Prior's concern with the syntactic decomposition of English modal sentences into "mode" and "dictum" leads him to investigate the extent to which an English "subordinate clause" may be transformed without a change in its meaning, a problem that was later famously treated by R. Montague in his formal generation of English relative clauses (Thomason 1974). Also, some of the remarks in "The Craft" anticipate the arguments for preferring intensional over extensional logic in projects like the Montague grammar or Bressan's axiomatisation of classical mechanics (Bressan 1972).

1. Modality as quantification over possible worlds

After showing in the first 15 pages of Chapter 1, Part V, different ways in which modal symbols resemble quantifiers, Prior opens Section 3 (Modality as a Form of Quantity) with an explanation for the similarity in their behaviour. There he introduces the idea of possible worlds for the first time. It may be, he speculates, that modal signs and quantity signs are both quantifiers, but they measure out different things, with the "signs of modality... operating upon a peculiar subject-matter, namely possible states of affairs" (p. 736). He illustrates his idea by interpreting an English sentence using both types of quantifiers.

(1) Something is able to write while not writing.

We can understand sentence (1) in two ways:

- (1a) There is something of which it is true that is not-writing in the actual state of affairs, and there is a possible state of affairs in which it would be writing.
- (1b) There is something of which it is true that there is some possible state of affairs in which it would be both writing and not-writing (p.737).

It can easily be shown that the use of "possible states of affairs" here is identical to what will later become the standard way in possible world semantics for natural languages. We can formalise both readings using x as a variable for individual objects, w as a variable for possible states of affairs, a as a constant designating the actual state of affairs, and the predicate constants writing and not-writing to stand for the respective English verb phrases. Then a) and b) become a') and b') below:

- (1a') $\exists x [\text{not-writing}(x, a) \ \& \ \exists w [\text{writing}(x, w)]]$; and
- (1b') $\exists x \exists w [\text{writing}(x, w) \ \& \ \text{not-writing}(x, w)]$.

In the next paragraph Prior recognises another metaphor used in everyday language for the objects of modal quantification, - the word "chances". It is usual, he says, "to quantify over entities called chances, - for 'X cannot be Y' we often say 'There is no chance of X being Y'" (p. 737). He cites as sources for this idea Wallis's account of hypotheticals, Wittgenstein's account of logically necessary/impossible propositions via their truth values in every possible combination of truth values of their components, and Carnap's definition of necessity in terms of 'state-descriptions'.

Prior rejects the possibility that modalities can be completely absorbed by quantitative terms, because none of the theories he mentions manages to completely reduce "modality to quantity". His argument is that although we may think of "modal distinctions as distinctions in quantity" (p.737), the variables that modal quantifiers bind still carry modal meaning. The variables stand for the following entities: "possibilities", "chances", "possible states of affairs", "possible combinations of truth-values", etc.

Prior then examines other possible explanations for the similarity between modalities and quantifiers, (found in the theories of Whately and Aldrich), presents other theories that attempt to explain modality using extensional means (especially the theory of Anderson), and systematically dismisses them on the grounds that they end up trivialising modal language: all truths become necessary truths. (p. 743) He concludes by making his point once again: the only way to avoid trivialisation of this kind is to interpret modal operators as quantifiers over modal objects, i.e. to allow that "modal predicates are quantifiers operating upon entities which already have a modal character (possible states of affaires, chances, etc.)" (p. 744).

2. Decomposition of modal propositions and the extraction of the "dictum"

Prior starts the discussion of modal logic on p. 721 of "The Craft" with praises for Aristotle's technique of decomposing natural language sentences. A modal sentence is rephrased into a "subordinate clause" that expresses the proposition, to which modalities can be attached as predicates. Later Aristotelians present this split as "dictum" and "mode". Prior finds the move "ingenious" because it makes the otherwise implicit role of the modal words explicit. The move is not intuitive at all, because sentences are finite linguistic forms and are difficult to dissect. Prior illustrates this with a simple modal sentence in English.

(2) It may be so.

The extraction of the "subordinate clause" from (2) produces the proposition "that it should be so". Applying the modal predicate back to it gives us a version of the initial sentence that is much easier to analyse.

(2') That it should be so *is possible*.

The reshuffling has turned the proposition (the part behind that) into "a subject" of a property. It is easy now to manipulate modal operators, quantifiers and the negation around propositions and play with their combinations. On the pages that follow, Prior uses a similar approach to turn sentences into predicates. To illustrate in English the medieval distinction between propositions *in sensu diviso* and *in sensu composito*, Prior reduces the sentences under consideration to complex predicates: "that he should write while he is not writing"; "x is able to write-while-not-writing", "whatever is not writing, is necessarily not writing". More than a decade later, Montague defined a rigorous procedure for performing such transformations in his famous formal fragments of English (Thomason 1974). The syntactic move known as "Quantifying-in" rule, that generates relative clauses of the type "x such that p", made a real breakthrough in the formal semantics of natural languages.

3. Intensional predicates and properties

Intensional predication is a theme that Prior often touches upon, never dealing with it explicitly and separately, and only within discussions on other related topics. But these are topics about the fundamental significance and understanding of modality: Can modal logic be reduced to extensional logic, or it should be the other way around? Prior's inclination towards the second option is evident not only in his discussion on possible worlds, or on the syntactic advantages of doing modal logic (p. 747-8), but also in the discussion of properties. "Every table here is light brown in colour" is neither a logically nor a physically necessary proposition, says Prior, but "Every table here is a table" would be logically necessary, while saying of objects of certain constitution that they reflect a certain light colour would be physically necessary (p. 738). Medieval logicians recognised this difference in attribution (natural, removed and contingent) and Prior acknowledges that assuming it allows some common sense entailments. Thus, when a predicate expresses "a property" Y of a subject X, then "Every X is Y" is always true, but when the predicate Y is not a property expressing the nature of the subject X, then "Every X is Y" is a contingent proposition (p. 739). Here again we see the anticipation of an important area of study in modal logic that will soon preoccupy many logicians. In 1960s, Richard Montague published three seminal papers on the formalisation of English where he showed that intensional contexts in natural languages can be successfully analysed if grammatical categories are treated as modal functions. In the early 1970s another important work in modal logic that was based on intensional predicates was published: Aldo Bressan's *Interpreted Modal Calculus*. In the attempt to express classical mechanics in axiomatic form and avoid some previously noticed problems that occur when this is done in ordinary predicate logic, Bressan constructed a powerful modal language based on an elaborate theory of modal attributes. Approximately 20 years after the "The Craft", Bressan showed what Prior suspected right from the start, that if taken as basic, modal predication boosts the expressive power of formal logic.

References:

Bressan, A., *General Interpreted Modal Calculus*, Yale University Press, 1972.

Copeland, B. J., "Prior's Life and Legacy" in B. J. Copeland (ed.) *Logic and Reality*, Oxford: Oxford University Press, 1996.

Copeland, B. J., "Genesis of Possible World Semantics", *Journal of Philosophical Logic*, Vol. 31, No. 2 (Apr. 2002), pp. 99-137.

Geach, P. T. and Kenny, A. J. P., "Introduction" to A. N. Prior, *The Doctrine of Propositions and Terms*, edited by P. T. Geach and A. J. P. Kenny, London: Duckworth, 1976.

Prior, A. N., *Formal Logic*, Oxford: Clarendon Press, 1955.

Prior, A. N., "The Craft of Formal Logic", unpublished manuscript (1950-51) held in the Bodleian Library.

Thomason, R. H., (ed.), *Formal Philosophy: Selected Papers of Richard Montague*, New Haven and London: Yale University Press, 1974.