# Time, chance and the quantum

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**Abstract**

I discuss the meaning of probability in the Everett-Wheeler interpretation of quantum mechanics, together with the problem of defining histories. To resolve these, I propose an understanding of probability arising from a form of temporal logic: the probability of a future-tense proposition is identified with its truth value in a many-valued and context-dependent logic. In short, probability is degree of truth. These ideas appear to be new (though I expect correction on this), but they are natural and intuitive, and relate to traditional naive ideas of time and chance. Indeed, I argue that Everettian quantum mechanics is the only form of scientific theory that truly incorporates the perception that the future is open.

It was with surprise strongly flavoured by intrigue that I received an email "reminding" me that a call is open for papers at the Arthur Prior Centenary Conference. Surprise because I am no philosopher, certainly no logician, and would not know how to present a professionally acceptable paper at this conference. But also intrigue becaue in my own professional life as a quantum theorist I have been brought to appreciate the importance of temporal logic in an understanding of quantum theory. The forms of temporal logic which seem to me to be needed are not, as far as I can see, quite the same as those of Prior and other logicians such as Belnap; it would indeed be an intriguing prospect if I could present these ideas to philosophers and ask what they make of them. This sense of intrigue has led me to extract some passages from the published versions of talks I have given to physicists and others [9, 10, 11], and present them in the form of this abstract: I am intrigued to know what response it will get. If this should be seen as a cheap and naughty way of getting an education from your referee, I can only apologise.

## PROBABILITY AND THE FUTURE

Within the Everett-Wheeler (EW) understanding of quantum mechanics, what kinds of statements can be made at time $t$ about some future time $s > t$, if the universal state vector is known to be $|\Psi(t)\rangle$ and its decomposition with respect to experience states of a particular observer is $\sum_n |\eta_n\rangle|\Phi_n(t)\rangle$? From the external perspective, the future state $|\Psi(s)\rangle$ is determined by the Schrödinger equation and there is no question of any probability. From the internal perspective relative to an experience state $|\eta_n\rangle$, there is a range of possible future states $|\eta_m\rangle$, and probabilities must enter into the statement of what the future state will be. But here is a fundamental problem: there is *no such thing* as what the future state will be. As Bell pointed out [3], there is no connection between a component of $|\Psi\rangle$ at one time and any component at another time; so what is it that we can assign probabilities to? How can "the probability that my state will be $|\eta_m\rangle$ tomorrow" mean anything when "my state will be $|\eta_m\rangle$ tomorrow" has no meaning?

## THE CLASSICAL FUTURE

This puzzle takes us back to ways of thinking that are much older than quantum mechanics, indeed older than all of modern science. The success of Newtonian deterministic physics has led us to assume that there always is a definite future, and even when we drop determinism we tend to continue in the same assumption. There is a future, even if we do not and cannot know what it will be. But this was not what Aristotle believed, and maybe it is not what we believed when we were children.

Aristotle, in a famous passage [1], considered the proposition "There will be a sea-battle tomorrow". He argued that this proposition is neither true nor false (otherwise we are forced into fatalism). Thus he rejected the law of excluded middle for future-tense statements, implying that they obey a many-valued logic. Modern logicians [7] have considered the possibility of a third truth-value in addition to "true" or "false", namely $u$ for "undetermined", for future-tense statements. But, interestingly, Aristotle admitted that the sea-battle might be more or less likely to take place. This suggests that the additional truth values needed for future-tense statements are not limited to one, $u$, but can be any real number between 0 and 1 and should be identified with the probability that the statement will come true. Turning this round gives us an objective form of probability which applies to future events, or to propositions in the future tense; in a slogan,

$$\text{Probability} = \text{degree of truth.}$$

This translation of Aristotle's position seems so natural that it has surely been developed already. However, I have been unable to find it in the literature of probability theory, temporal logic or many-valued logic. In an early paper [5] (earlier than what are generally regarded as the first papers on many-valued logic) Łukasiewicz introduced truth values between 0 and 1 and equated them with probabilities, but in a different sense from that of quantum mechanics (he was searching for a notion of objective probability, but found it only in propositions containing a free variable; he rejected any application to future-tense statements with no free variables, to which he thought probability did not apply because at this time (1913) he believed in determinism). The notion of "degree of truth" occurs in fuzzy logic and philosophical discussions of vagueness (for a critical account see [12]), but not, as far as I can see, in the philosophy of probability. Maybe this is because it requires modifications to some of the usual properties of truth values.

First, truth values are usually assumed to have the property that logical connectives like $\wedge$ (and) and $\vee$ (or) are "truth-functional", i.e. the truth values of $p \wedge q$ and $p \vee q$ are determined by those of $p$ and $q$. The probability of a compound statement, however, is constrained but not determined by the probabilities of its constituents: for the probabilities of $p \wedge q$ and $p \vee q$ we only have inequalities

$$\begin{aligned} &P(p), P(q) \geq P(p \wedge q) \geq 1 - P(p) - P(q), \\ &P(p), P(q) \leq P(p \vee q) \leq P(p) + P(q). \end{aligned}$$

Although degrees of truth for vague statements are usually assumed to be truth-functional, some authors have argued that they should behave like probabilities, as above. ([4]; see also [12]).

A more radical departure from the usual properties of truth values is that the probability of a proposition referring to a future time $s$ depends not only on the proposition itself (taking the time $s$ to be a part of the proposition), but also on the time $t$ at which the proposition is considered. For an observer experiencing the state $|\eta_n\rangle$ at time $t$, the probability of experiencing $\eta_m\rangle$ at a future time $s$ is

$$\frac{\left|\left(\langle\eta_m|\langle\Phi_m(s)|\right) \mathrm{e}^{-iH(s-t)/\hbar} \left(|\eta_n\rangle|\Phi_n(t)\rangle\right)\right|^2}{\langle\Phi_m(s)|\Phi_m(s)\rangle\langle\Phi_n(t)|\Phi_n(t)\rangle}. \tag{1}$$

Thus our account of probability requires that the truth value of the proposition "My experience at time $s$ will be $|\eta_m\rangle$" depends on the time at which the proposition is uttered. Such dependence on a context of utterance is nothing new (consider the truth value of the proposition "it is raining"), but that can usually be understood as being because the context of utterance is needed to fill in an incomplete proposition (in our example, it provides the time and place at which it is raining). There is no question of this in the case of the occurrence of $|\eta_m\rangle$ at time $s$.

## THE TRUTH OF THE PAST

The classical position (in classical philosophy, if not classical physics) would be that propositions referring to the past and present are either true or false; it is only the future that is uncertain. Thus if $P(s,t)$ is the truth value at $t$ of a proposition referring to time $s$, we should have

$$\begin{aligned} 0 \leq P(s,t) \leq 1 \quad & \text{if } s > t, \\ P(s,t) = 0 \text{ or } 1 \quad & \text{if } s \leq t. \end{aligned} \tag{2}$$

It is not clear whether this can or should be maintained in quantum theory. Bell's point [3] about the lack of connection between experience states at different times applies to the past as much as the future. We do have an empirical warrant for our past states, as we do not for future states, in the form of memory, which is not symmetric under time reversal. This has yet to be modelled quantum-mechanically, but hopefully it can be shown that there is a physical process which leaves a record in the state at one time of a sequence of states in the past, and that this record is consistent with the probabilities (1). Maybe it would go further and establish transition probabilities such as have been postulated for modal interpretations [2, 8]. If so, this might provide justification for truth values satisfying (2).

Even if this could be established as a feature of quantum systems with memory, the theory would still be vulnerable to Bell's charge of temporal solipsism: a memory state is still a present state, and does not constitute a genuine past. Markosian [6] has pointed out that this openness of the past is an inevitable consequence of time-reversal symmetry in a theory with an open future.

## THE OPEN FUTURE

We find it hard, in a scientific theory, to accommodate the idea that there is no definite future. To be sure, we have indeterministic theories in which the future is not uniquely determined by the past, but such stochastic theories deal with complete histories encompassing past, present and future; probabilities refer to which of these histories is actualised. Indeterminism consists of the fact that there are many such histories containing a given past up to a certain time, so the future extension is not unique; but the underlying assumption is that only one of these future histories is real, so that the future is fixed even though it is not determined. In contrast, the EW formulation of quantum mechanics (or, as Bell [3] calls it, the "Everett (?) theory") is the only scientific theory in which the future is open. This is not (as Bell seemed to think) a problem for the theory; it tells a truth which we should recognise.

# References


[1] Aristotle. *De Interpretatione*. Peripatetic Press, Grinnell, Iowa, 1980. Chap. 9.

[2] G. Bacciagaluppi and M. Dickson. Dynamics for density operator interpretations of quantum theory. *Found. Phys.*, 29:1165, 1999. arXiv:quant-ph/9711048.

[3] J. S. Bell. Quantum mechanics for cosmologists. In C. Isham, R. Penrose, and D. Sciama, editors, *Quantum Gravity 2*, pages 611–637. Clarendon Press, Oxford, 1981.

[4] D. Edgington. Validity, uncertainty and vagueness. *Analysis*, 52:193–204, 1992.

[5] J. Łukasiewicz. Logical foundations of probability theory. In L. Borkowski, editor, *Selected Works*, pages 16–63. North-Holland, Amsterdam, 1970. First published in Cracow, 1913.

[6] N. Markosian. The open past. *Philosophical Studies*, 79:95–105, 1995.

[7] A. Prior. *Past, Present and Future*. Clarendon Press, Oxford, 1967.

[8] A. Sudbery. Diese verdammte Quantenspringerei. *Stud. Hist. Phil. Mod. Phys.*, 33B:387–411, 2002. arXiv:quant-ph/0011082.

[9] A. Sudbery. The Everett-Wheeler interpretation and the open future. In G. Jaeger, A. Khrennikov, M. Schlosshauer, and G. Weihs, editors, *Advances in Quantum Theory*, pages 492–497. AIP, 2011.

[10] A. Sudbery. Philosophical lessons of entanglement. In D. Home, G. Kar, and A. S. Majumdar, editors, *75 Years of Quantum Entanglement: Foundations and Information Theoretic Applications*. AIP, 2011.

[11] Anthony Sudbery. Einstein and Tagore, Newton and Blake, Everett and Bohr: the dual nature of reality. In P. Ghose, editor, *The nature of reality: the perennial debate*. Cambridge University Press, Delhi, to be published. arXiv:1205.1479.

[12] T. Williamson. *Vagueness*. Routledge, London, 1994.