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Priorean Temporal Operators in Natural Language

A. N. Prior's work influenced early theorizing on natural language tenses in the field of formal semantics[1]. During the seventies, several theorists working in the tradition of intensional logic endorsed the view that tenses may be formalized as sentential temporal operators. The most influential among them was Montague[2]. In PTQ, Montague analyzed the present perfect and the simple future of English by using such operators[3]. His PTQ formal system translates sentences **(1)** and **(2)** into the temporal-logic language chosen by Montague as **(3)** and **(4)**, respectively. The semantics of the operators H and W is given by a rule that determines the extension of formulas of the form $H\phi$ and $W\phi$ relative to a PTQ model $\Re$, a world i, a time j, and an assignment function g –all of them taken from $\Re$[4].

(1) John has run

(2) Mary will talk

(3) H **run**(j)

(4) W **talk**(m)

$[H\phi]^{\Re, i, j, g} = 1$ iff there is a time $j' \in J$ such that $j' \neq j, j' \leq j$, and $[\phi]^{\Re, i, j', g} = 1$

$[W\phi]^{\Re, i, j, g} = 1$ iff there is a time $j' \in J$ such that $j' \neq j, j \leq j'$, and $[\phi]^{\Re, i, j', g} = 1$

The extension of $H\phi$ and $W\phi$ relative to $\Re$, i, j, g is determined by the intension assigned to ϕ relative to $\Re$, g. Such an intension is a function from world/time pairs –taken from $I \times J$– to the truth-values truth and false.

Let us call SOAT (for standard operator account of tenses) the analysis of natural language tenses proposed in PTQ. Prior himself did not favor the model-theoretic approach to temporal-logic systems that Montague and other formal semanticists adopted in their writings. He always preferred a proof-theoretic approach, even after the development of possible world semantics in the early sixties. Prior was willing to accept instants of time as individual entities –and a relation of temporal order defined on them– only to the extent to

[1] Prior's contributions to temporal logic can be found in his 1957, 1967, and 1968.
[2] But see also Kamp 1971, pp. 228-232, Kaplan 1977, sections VI. (i) and XVIII, and Lewis 1980, sections 5 and 8. The assumption that tenses can be plausibly treated as temporal operators is at the hearth of Kaplan and Lewis's operator argument for temporalism. For critical discussion of this argument, see King 2003 and Cappelen and Hawthorne 2009, chapter 3.
[3] Montague 1973, pp. 252-253, fn 6, 257-259. See also Dowty *et al.* (1981), chapter 5, and chapter 7, section VIII.
[4] A model of PTQ is a quintuple $\langle A, I, J, \leq, F\rangle$, where A, I, and J are non-empty sets of inidividuals, worlds, and times, respectively, $\leq$ is a linear order on J, and F is a function assigning appropriate intensions to the constants of the interpreted language (see Montague 1973, pp. 257-260).

which they could be constructed out of tensed facts[5]. That being said, it is worth noting that SOAT vindicates two theoretical insights of Prior: (i) the idea that temporal talk can be analyzed as involving temporal operators; and (ii) the idea that there are temporally neutral contents (temporalism)[6].

Despite the influence of Prior and Montague's pioneering work, the following decades were characterized by a paradigm shift in the study of tenses. This shift can be traced back to two seminal papers: [Kamp 1971] and [Partee 1973]. Linguists progressively gathered a powerful body of data against SOAT and other operator-based accounts of tense. The data showed that these accounts fail to predict the right truth-conditions for various types of natural-language sentences. The alternative accounts of the data proposed by most theorists working in this area postulate covert variables ranging over times or events at the level of logical form. As a result, most formal semanticists nowadays believe that the accounts of tense constructed along the lines of the operator paradigm are inferior to the variable-based accounts that have been proposed in the literature.

Below I provide a list of the kind of constructions that have been discussed in the formal semantic literature on tenses. Each heading is accompanied by one or more prototypical examples of the corresponding type of sentence. Most of the examples have at least one reading that SOAT either cannot predict or can only predict by making controversial assumptions about the logical form of the relevant sentence.

Kamp/Vlach sentences [Kamp 1971, Vlach 1973]

(5) A child was born who will be king
(6) One day, all persons alive then would be dead

Pronoun-like uses of tenses [Partee 1973, 1984, Kratzer 1998]

(7) I didn't turn off the stove
(8) Sheila had a party last Friday and Sam got drunk
(9) Whenever Mary telephoned, Sam was asleep

Time adverb sentences [Dowty 1982]

(10) John left yesterday

Enç sentences [Enç 1986]

(11) Every fugitive is now in jail

Embedded tenses [Enç 1987, Ogihara 1996, Abusch 1997, Kusumoto 2005]

(12) John heard that Mary was pregnant
(13) Hilary married a man who became president of the U. S.

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5 See e. g. Prior 1968, paper XI, and Prior and Fine 1977, p. 37. See also the discussion in Prior and Fine 1977, chapter 8.

6 In PTQ, the temporally neutral contents are the intensions.

Even though sentences like **(5)-(13)** reveal the inadequacy of SOAT, we must not conclude that the desired readings of these sentences cannot be obtained by using temporal operators. A striking aspect of the linguistic literature on tenses that I have briefly reviewed here is that the operator paradigm has often been discarded in the light of some piece (or pieces) of evidence without an exhaustive examination of the ways in which the target version of the paradigm –which typically is SOAT– could be amended. However, one can find valuable suggestions as to how to improve SOAT both in the writings of Prior and in the writings of the authors who have been inspired by his work. In this 'Priorean literature' various aspects of SOAT have been revised. Let me summarize some of these developments.

(I) *Interval sensitivity:* The semantic clauses for the PTQ operators *H* and *W* quantify unrestrictedly over the times that precede or follow a given evaluation time. In the formal language of PTQ there is no way of assessing a formula with respect to a specific past or future time interval.

But Prior's metric tense-logic allows for this possibility[7]. On this system, the past and future operators are treated as dyadic operators containing an argument place for an interval measure and an argument place for a sentential formula. These operators give rise to formulas of the form $Pn\phi$ and $Fn\phi$, which may be read, respectively, as 'it was the case *n*-much ago that ϕ' and 'it will be the case *n*-much hence that ϕ'[8]. One can use formulas of this kind to get the right readings for sentences like **(7)** and **(8)**. In a recent paper devoted to metric tense-logic, Cressswell (2013) shows that the Kamp/Vlach sentences **(5)** and **(6)** can also be formalized by using the dyadic operators P and F.

There are other ways of introducing interval-sensitivity into a temporal logic framework. In a series of papers, Blackburn has proposed a number of extensions of Priorean temporal logic in which new sorts of atomic propositional symbols are introduced[9]. These symbols are true at specific time instants or time intervals. They are the temporal logic analogues of temporal variables, temporal indexicals, and calendar terms.

(II) *Double-indexing:* SOAT is a single-index theory. Kamp (1971) famously argued for double-indexing by considering example **(5)**. He symbolized **(5)** as **(14)** –where P, F, and N are monadic sentential operators.

(14) $P\,\exists x$ (x is a child $\wedge$ x is born $\wedge$ $N\,F\,x$ is king)

In Kamp's 1971 system, sentential formulas are true or false relative to a time of utterance and a time of evaluation[10]. His 'now'-operator N 'checks' whether the operated formula ϕ is true when its evaluation time is the time of utterance. The past and future operators P and F shift the time of evaluation in the standard way. Kamp's double-indexed treatment of these three operators is essentially the one presented in Kaplan's *LD* formal system (see Kaplan 1977, section XVIII). Dowty (1982) refined Kamp's double-index approach by defining past, present, and future temporal operators that relate two temporal indices: the time of utterance and another time playing a role analogous to Reichenbach's (1947, §51)

[7] Metric tense-logic is discussed in Prior 1957, chapter 2, pp. 11-15, 1967, chapter 6, and 1968, paper IX.

[8] The interval-measure term appearing in such formulas can be quantified. This operation produces formulas of the form $QnPn\phi$ and $QnFn\phi$, where Q is a monadic quantifier

[9] See e.g. Blackburn 1993, 1994, and 2006.

[10] Truth-value assignments are also relative to an interpretation $\mathfrak{R}$ and a pair J consisting of a set of times T and a partial ordering < (see Kamp 1971, §1).

point of reference. By relying on these operators, Dowty proposed specific formalizations for the English constructions illustrated in **(5)**, **(7)**, **(8)**, **(10)**, and **(12)**.

(III) *Subsententiality:* The operators *H* and *W* of PTQ are sentential. They operate on sentential formulas. However, one may postulate temporal operators at the subsentential level. Such operators might be characterized as predicate operators, or maybe as operators that act on sub-predicative semantic constituents. This seems to be a natural view to adopt given the fact that in English surface syntax tenses modify verbs, or verb lexemes, rather than whole sentences. The possibility of a subsentential theory of temporal operators has been envisaged by Enç (1986, pp. 421-422) and Salmon (1989, p. 386, fn 31). But they did not explore this theoretical possibility in detail[11].

In my paper for the ANP2014 I will discuss some of the Priorean developments sketched in the previous paragraphs. In a nutshell, I shall argue that that a sophisticated operator-based account of tenses must incorporate the features **(I)-(III)** in order to overcome the challenges posed by sentences like **(5)-(13)**.

As I mentioned above, Prior devised a logical formalism that exhibits **(I)** –namely, metric tense-logic. To my knowledge, the most sophisticated operator account of tenses that incorporates feature **(II)** is the one developed in [Dowty 1982]. The account proposed by Dowty in that paper might seem odd to a Prior scholar. For Dowty's system exploits the Reichenbachian distinction between point of speech and point of reference; a distinction that Prior famously criticized (see Prior 1967, pp. 12-15). I will analyze the arguments given by Prior against Reichenbach's theory. In the context of Dowty's theory, I shall argue, one does not need to posit more than two temporal indices in order to account for the past perfect, the future perfect, or the future perfect progressive –which is the tense mentioned by Prior in his criticism (p. 13). The distinction between point of speech and point of reference is required in such a theory in order to account for the truth-conditions of sentences like **(5)** and **(6)**. In p. 14 Prior discusses the sequence of tense. He notes that a sequence-of-tense rule is applied in English when speakers want to convey what nowadays we call the simultaneous reading of a sentence like **(12)**. But, arguably, there are ways of predicting this reading within a double-index theory.

In the Priorean literature, feature **(III)** has remained virtually unexplored. I shall outline an account that preserves the basic semantic framework of Dowty's theory but treats tenses as non-sentential operators. On this account, temporal operators act on a certain class of pre-predicative syntactic constituents. I call such constituents *radicals*. Let me illustrate the account I have in mind by interpreting an English sentence in the simple past.

Syntactically, radicals are expressions that give rise to predicates when combined with temporal operators. Semantically, they get standard predicate extensions relative to worlds and time intervals. The interpretation of an *n*-ary radical $\varphi\ x_1, \ldots x_n$ is a function mapping world/time-interval pairs onto sets of *n*-tuples of individuals. I assume that the past operator PAST is context-sensitive in two ways. It requires two different time intervals *t* and *t*' that must be contextually provided. Think of *t* as the time of utterance and think of *t*' as a contextual point of reference. A past temporal operator –such as PAST– requires that $t' < t$. In other words, when a past operator is used, *t*' is a point of reference lying to the past of *t*. Let W, T, and I respectively be the set of all worlds, the set of all time intervals, and the set

[11] The question of whether there are sub-sentential intensional operators in natural language has been discussed in the philosophical literature on semantic relativism (see Cappelen and Hawthorne 2009, pp. 74-76, and Kölbel 2011, pp. 144-145).

of all individuals. Let $\supset$ be the parthood relation for time intervals. Finally, let $<$ be the earlier-later relation between time intervals[12]. Given an n-ary radical $\varphi\ x_1, \ldots x_n$, PAST is defined as follows:

> $|\mathbf{PAST}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: \mathrm{W} \times \mathrm{T} \times \mathrm{T} \rightarrow \wp(\mathrm{I}_1 \times_{\ldots} \times \mathrm{I}_n), f(\langle w, t, t' \rangle) = \{\langle i_1, \ldots i_n \rangle$: there is a time t'' such that $t'' < t$, $t'' \supset t'$, and $\langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$

$|\mathrm{PAST}\varphi\ x_1, \ldots x_n|$ is a function that maps any given $\langle w, t, t' \rangle$ triple onto the set of those n-tuples $\langle i_1, \ldots i_n \rangle$ that appear in the range of the function $|\varphi\ x_1, \ldots x_n|$ when a time t'' lying to the past of t, and mereologically contained in t', is given as input to that function.

Let FROWN be the radical associated with the English verb 'frown'. Given an appropriate semantic clause for FROWN, we can interpret the predicate PASTFROWN x.

> $|\mathbf{FROWN}\ \boldsymbol{x}| = f: \mathrm{W} \times \mathrm{T} \rightarrow \wp(\mathrm{I}), f(\langle w, t \rangle) = \{i$: i frowns in w at $t\}$

> $|\mathbf{PASTFROWN}\ \boldsymbol{x}| = f: \mathrm{W} \times \mathrm{T} \times \mathrm{T} \rightarrow \wp(\mathrm{I}), f(\langle w, t, t' \rangle) = \{i$: i frowns in w at some time t'' such that that $t'' < t$ and $t'' \supset t'\}$

The interpretation of the monadic predicate PASTFROWN x is a function mapping any given $\langle w, t, t' \rangle$ triple onto the set of individuals that frown at some time preceding t –the time of utterance– and contained in t' –the reference time–.

A sentence in the simple past, such as "John frowned", can now be truth-conditionally assessed, provided that the relevant context of utterance (let us call it c) has determined a world of utterance w_c, a time of utterance t_c, and a reference time $t_{\mathbf{FROWN}(c)}$. If, for example, the speaker of c is intuitively talking about what John did on October the 10th, 2013, then $t_{\mathbf{FROWN}(c)}$ = October 10, 2013. Here is the truth-condition of "John frowned" relative to c.

> **PASTFROWN** ***John*** is true in c iff John frowns in w_c at some time t such that $t < t_c$ and $t \supset t_{\mathrm{FROWN}(c)}$.

It is easy to see how the past perfect can be treated in this system. Assuming that $t' < t$, we can define a radical operator that looks for predicate extensions at times lying to the past of t', rather than contained in t'.

> $|\mathbf{PASTPERF}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: \mathrm{W} \times \mathrm{T} \times \mathrm{T} \rightarrow \wp(\mathrm{I}_1 \times_{\ldots} \times \mathrm{I}_n), f(\langle w, t, t' \rangle) = \{\langle i_1, \ldots i_n \rangle$: there is a time t'' such that $t'' < t$, $t'' < t'$, and $\langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$

Unlike PAST and PASTPERF, future radical operators require that $t < t'$. That is to say, the reference time of such an operator must be in the future relative to the time of utterance. FUT and FUTPERF and are the radical operators corresponding to the simple future and the future perfect, respectively.

> $|\mathbf{FUT}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: \mathrm{W} \times \mathrm{T} \times \mathrm{T} \rightarrow \wp(\mathrm{I}_1 \times_{\ldots} \times \mathrm{I}_n), f(\langle w, t, t' \rangle) = \{\langle i_1, \ldots i_n \rangle$: there is a time t'' such that $t < t''$, $t'' \supset t'$, and $\langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$

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[12] Given two intervals t and t', $t < t'$ iff every time instant included in t precedes every time instant included in t'. Intervals are sets of instants. For a definition of the notion of interval, see Ogihara 1996, p. 24.

$|\mathbf{FUTPERF}\varphi\ \boldsymbol{x_1, \ldots x_n}| = f: \mathrm{W} \times \mathrm{T} \times \mathrm{T} \rightarrow \wp(\mathrm{I}_1 \times_{\ldots} \times \mathrm{I}_n), f(\langle w, t, t' \rangle) = \{\langle i_1, \ldots i_n \rangle$: there is a time t'' such that $t < t''$, $t'' < t'$, and $\langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$

As I mentioned above, Dowty proposed a double-indexed operator-based account to formalize sentences such as **(5)**, **(7)**, **(8)**, **(10)**, and **(12)**. By modeling English tenses as radical operators –I will argue in my paper– we can also predict the desired truth-conditions for sentences like **(10)**, **(11)**, and **(13)**. These are sentences that Dowty's 1982 theory cannot account for in a straightforward way.

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