Priorean Tense Logic, Tense Predicate Logic and many-sorted Predicate Logic

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Abstract

1. Historical context.

One of the greatest pieces of luck I have had in my life is that my arrival at UCLA in the fall of 1965 coincided with the visit that semester of Arthur Prior. He had come as a Visiting Professor in the Philosophy Department and I took one of the two courses he offered that semester, an Upper Division course on Tense Logic, in which he covered roughly the material that appeared shortly afterwards in the book *Past, Present and Future* (Prior (1967)). Prior's course changed the topic of my Ph.D. and my earliest work was dominated entirely by Tense Logic and, at least initially, by (what I took to be) Prior's research agenda. That early work includes on the one hand the introduction of the tense operators SINCE and UNTIL and their functional completeness and on the other the treatment of 'now' as a 1-place sentential operator.

As time went by I became convinced that, contrary to what I believed during my time at UCLA, Priorean Tense Logic is not a suitable instrument for the analysis of temporal reference in natural languages such as English, and my own methods for dealing with the problems of tense and aspect have strayed a long way from the Priorean model. But a logician's interest in Prior-type tense logics I never lost. And, as far as I can tell, hybrid logics, of the kind developed and advocated by Patrick Blackburn, remain a serious alternative to the formal methods that are commonly employed in linguistic theories of aspect and temporal reference (Blackburn (1990)), (Blackburn (2006)).

When Prior came to UCLA, Cocchiarella had just completed his Ph.D. dissertation under Montague, in which he develops the syntax and model theory of systems of Tense Predicate Logic (Cocchiarella (1965)). (Cocchiarella came down to UCLA to defend his dissertation that fall.) I believe that for Prior this was the first close confrontation with the use of model theory in the development of temporal logics. My vague impression has always been that while seeing the formal merits and potentials of this approach, he never seriously considered it as an alternative to his own method, perhaps because he felt that the model-theoretic method begs the fundamental metaphysical issues that were the driving force behind his formal logical work and from which (I think he thought) formal methods could not and should never by divorced.

In fact, Prior had developed, in his work on the 'UT-calculus', a quite close alternative to the model-theoretic approach (Prior (1968*a*)). The first part of this talk will have a look at the relation between these two.

2. Model-theoretic Semantics and Prior's Logic of Earlier and Later

Since Montague succeeded, in the second half of the sixties, to show that the model-theoretic method could be applied with great profit to the analysis of meaning in natural languages, the use of model theory in linguistic semantics has been ubiquitous. But it has also for the most part been innocent of reflection on its logical foundations. In particular, the following formal presuppositions of the method are often ignored.

In the form in which Montague advocated model-theoretic semantics (Thomason (1974)), and in which it has usually been practiced ever since, there are strictly speaking two formalisms involved in each application. The first is the formalism used to identify the meanings of the expressions of the *Object Language* in the given application. I will refer to below as 'OL' – in the typical case OL is some fragment of some natural language – and to the formalism that is used to identify the semantic values of the expressions of OL as 'LFF' (for 'Logical Form Formalism'). Montague used his H(igher) O(rder) I(ntensional) L(ogic) as LFF; HOIL (or some close variant of it) has also been the formalism used in most other work in model-theoretic semantics.)

A thorough formalisation of a model-theoretic treatment of some OL, however, also involves a second, more comprehensive formalism, in which the both syntax of OL and its model theory can be explicitly formalised and within which all the relevant statement the treatment makes about OL should be formally deducible. (Among the statements that ought to be formally deducible are those to the effect that a certain expression E of OL gets in a model M the value denoted in M by a certain term τ of LFF.) Let us call this second formalism 'GFE', for 'General Formalisation Environment. GFE has to be at least as powerful as LFF, since it must be possible to develop the syntax and model theory of LFF within it. But in addition it must be possible to define within GFE the class of models postulated in the given application and GFE must be able to express quantification over the models belonging to this class (which is needed, for instance, in the standard semantic definition of logical consequence). All this means that in non-trivial applications of the model-theoretic method GFE has to be very powerful indeed. And in principle it is possible to ask, in connection with any such application, what a minimal GFE would be like that could do the job.

Normally this question isn't asked. In fact, the preconditions for asking aren't fulfilled because the application doesn't even attempt the degree of formalisation that would require an explicit specification of GFE. (For instance, what may be considered the core of any model-theoretic treatment, viz. the definition of 'semantic value' – the definition which specifies, typically by recursion on syntactic complexity in OL, what the semantic value τ is of any syntactically well-formed OL-expression E in any model M – will almost without exception be stated in informal mathematics. I take it that most formal logicians would, when pressed for such a formalisation, assume that it should be carried out within some first order theory of sets, such as ZF with Urelements. But adopting such a formal environment comes with heavy ontological commitments, both with regard to the needed Urelements and the set-theoretical superstructure. Once the problem of a thoroughly formalised model-theoretic semantics has been explicitly acknowledged, the question whether something more modest could not be used instead of GFE is hard to ignore.

It is in this light that the model-theoretic approach – exemplified in Montague's work on the semantics of natural language and in Cocchiarella's thesis – should be compared with the way in which Prior correlates in his 'Tense Logic and the Logic of Earlier and Later' (TL&LEL) 'Logical Forms' of his UT-calculus with the formulas of his P,F-calculus. Note in this connection that the core of any model-theoretic treatment invariably consists in its assignment of expressions of its LFF as Logical Forms to expressions of its OL. This is what model-theoretic semantics and the 'deduction-based semantics' of Prior's TL&LEL have in common: in both it is the association of Logical Forms with OL expressions that is central. But Prior establishes his pairings in a formal setting that is based on much more modest assumptions, and his procedure achieves what ought to be the ultimate goal of all 'truly formal' semantics: the pairings of OL expressions with their LFs are all formally deducible from some given set of axioms.

3. Prospects for Upscaling the TL&LEL-approach?

In the talk I will present the main ideas of TL&LEL with special emphasis on the aspect of it that is highlighted under 2. and then address the question whether and how this method could be extended to Tense Predicate Logic and, perhaps, beyond. I will conclude by drawing attention to the pitfalls that may lay in wait for one tries to extend methods developed for Propositional Tense Logic to the level of Tense Predicate Logic. In that connection I will review the two main formal results my early work on Tense Logic, from the time when Prior came to be one of my two principal mentors, intellectual anchors and idols. On the one hand there is the functional completeness of the tense logic whose tense operators are the 2-place operators $SINCE$ and $UNTIL$ (Kamp (1968)): when time is order-complete, every first order topologically definable tense operator can be expressed in this logic. This result does not only hold for Propositional Tense Logic but also for Tense Predicate Logic. On the other hand there are the results connected with the 1-place tense operator N, which was intended as a formalisation of the adverb *now* as it is found in English (Prior (1968*b*), Kamp (1971)): When N is added to a system of Propositional Tense Logic such as Prior's P,F-calculus (but the result can be stated in considerably more general terms) then it is eliminable:

every formula of the calculus with N is logically equivalent to one without N. But this result does not transfer to the case of Tense Predicate Logic: some formulas of this calculus that contain N are not equivalent to any formula that does not contain N.

It is not immediately obvious why there should be this difference between the functional completeness of SINCE and UNTIL and the eliminability of N. (The proofs that establish these results are fairly complicated and it is not easy to draw a simple moral just by laying them side by side.) A closer comparison of these results will be the final concern of this talk.

References

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