Hybrid Logic and ‘Now’ (Extended Abstract)

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1 Introduction

Hans Kamp’s paper “Formal properties of ‘now’” (1971) launched the technical study of the logic of indexicals. Kamp’s methods, especially the use of two-dimensional semantics which this paper introduced, became highly influential. Arthur Prior was one of the first to respond, and in his paper “‘Now’” he explored Kamp’s work in detail.[1] Priors philosophical commentary drew on both English usage and the Castañeda (1967) analysis of the indexical ‘I’. His technical analysis ranged across two versions of his UT calculus, made a brief but crucial intermezzo into hybrid logic, and concluded with a detailed study of tense logic enriched with the universal modality $\Box$ and a Kamp-style ‘now’ operator J.

As this summary suggests, Prior’s discussion is both interesting and demanding; the paper ranges widely, and makes many detailed points. This abstract focuses on two, which we regard as central. First, Prior demonstrates that Kamp’s two-dimensional analysis is merely one approach among many: a one-dimensional approach is also possible—if we include an instant constant i in the UT calculus. Second, Prior argues that if we move into hybrid logic and turn this instant constant into a *propositional* constant n, it is easy to capture the logic of J in a way that reflects Castañeda’s central point. As we shall see, Prior was right. His hybrid logic for ‘now’ is indeed built squarely upon Castañeda’s work. Moreover, the logic *is* complete, as we shall show using a simple semantic argument.

2 In His Own Words

It is easy to see why Kamp’s work caught Prior’s attention: at first glance, it seems that Kamp’s demonstration that there is a genuine logic of ‘now’ contradicts Prior’s redundancy approach to the present. But this is mistaken. After discussing several examples, Prior’s reaction is typically robust:

[1] Prior’s paper was originally published as “‘Now’” in *Noûs*, 2:101–119, 1968. An addendum, correcting a technical glitch, appeared as “‘Now’, corrected and condensed”, in *Noûs*, 2:411–412, 1968. A combined version (in which Prior’s Polish notation was converted to standard notation) appeared under the title “‘Now’” in the new edition of *Papers on Time and Tense* (Prior; 1968/2003). In this abstract, all quotations from and page references to “‘Now’” are to this combined version.

> There is surely no need for prolonged agonising about all this. [... A]s far as English idiom goes, it seems clear that constructions involving the word 'present' fit a redundancy theory fairly well, that ones involving the word 'now' do not fit it at all well, and that ones involving the plain present tense or the plain 'it is the case that' are in between. (Prior; 1968/2003, p. 173)

But it is also clear that Kamp's work marks a major advance:

> [U]ntil recently I would have gone further than this, and said that the formalist not only *can* do without the idiomatic 'now' but *must* do without – that our ordinary use of now has a fundamental disorderliness about it only makes it unamenable to to formalisation [...] Recently, however, I have been convinced to the contrary by Hans Kamp [...] (Prior; 1968/2003, p. 174)

Prior also remarks that he formerly thought that adding 'now' to tense logic would have a "quite explosive effect" (1968/2003, p. 176). But while Kamp has shown otherwise, is Kamp's approach the only possible one?

Prior begins his investigations in the UT calculus. A key construction in the UT calculus is the expression $Ta(\varphi)$, that is, the formula φ holds at the instant named a. Prior initially follows Kamp into a two-dimensional analysis: he introduces a form $Tab(\varphi)$ which allows him to 'track' a second time coordinate. However, influenced by Castañeda's remarks on 'I' he pulls back: as he states on page 182, it is possible to stay in a one-dimensional format (that is, to stick with $Ta(\varphi)$) if one introduces a constant, say n, for a particular instant. He notes that this reduction to a one-dimensional format preserves Kamp's insights. He then claims that the reduction will lead to an easier axiomatization; in support of this he cites C.A. Meredith's work on the property calculus.[2] Prior then begins the intermezzo which shall occupy our attention: he switches from the UT calculus to hybrid logic. He introduces n as a special *propositional* constant, states a number of axioms, and remarks:

> The postulates RL (modified), L1–5 and A1.1–A5 with the definition of J, will I think yield all the theorems in J that we want; and the definition of 'now' as expressing contemporaneity with some unspecified proposition which is true only at the time of utterance nicely formalises Castañeda's explanation of the use of 'now' in oblique contexts (Prior; 1968/2003, p. 184)

Then, with the following (almost apologetic) words, he retreats from hybrid logic to a weaker system (tense logic enriched with $\Box$ and J). The hybrid excursion is over:

> It may be felt, however, that this system is too much of a hybrid between a UT calculus and a tense logic. (Prior; 1968/2003, p. 184)

Memorable words—and probably the first time the word 'hybrid' has been used in connection with such logics![3]

[2]Prior cites their joint paper Meredith and Prior (1965) which deals with axiomatisation, but notes that Meredith's work on the subject dates back until at least 1953.

[3]Prior did not speak of hybrid logic; that term only gained currency in the 1990s, long after Prior's death; the publication of Blackburn and Seligman (1995) was the baptismal event. Prior regarded hybrid logic as a part of tense logic, indeed it was the *third grade of tense logical involvement*, as he explained in his paper "Tense Logic and the Logic of Earlier and Later", which also can be found in (Prior; 1968/2003). As the previous quotation seems to suggest, Prior's views on hybrid logic (third grade tense logic) were somewhat equivocal; see Blackburn (2006) for further discussion.

But Prior's previous remark ("will I think yield all the theorems in J that we want") is essentially a modestly stated completeness claim. And, as we shall show, it is correct. So we will now dive back into the pool that Prior has just vacated and show how prescient he was.

3 A Hybrid Tense Logic for 'Now'

Hybrid tense logic is a simple extension of ordinary Priorean tense logic in which it is possible to refer to times. It does this using special propositional symbols which nowadays are called *nominals*. Nominals are true at one and only one time: they 'name' the time they are true at.[4]

We will work in the same hybrid logic that Prior uses in "'Now'", that is, a language built on a set Nom of nominals (typically written i, j and k) and a set Prop of ordinary propositional symbols (typically written p, q and r). As connectives we take some truth functionally adequate collection of boolean operators (here we'll choose $\neg$ and $\wedge$) and the symbols F, P, and $\Diamond$. Formulas are built as follows:

$$\varphi ::= a \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid P\varphi \mid F\varphi \mid \Diamond\varphi.$$

We define $G\varphi$ to be $\neg F \neg\varphi$, $H\varphi$ to be $\neg P \neg \varphi$ and $\Box\varphi$ to be $\neg\Diamond\neg\varphi$

Models $\mathfrak{M}$ are triples $(T, \prec, V)$. Think of T as a set of times and $\prec$ as the earlier-later relation. However we won't impose any conditions on $\prec$, such as transitivity, or irreflexivity, or linearity to make this relation more time-like; we are going to follow Prior and investigate the minimal logic.[5]

The valuation function V, takes all atomic symbols (that is, both nominals and ordinary propositional symbols) to subsets of points of T. Ordinary propositional symbols are unrestricted in their interpretation: they encode arbitrary information, such as when it was sunny in Masterton, or the timing of the All Blacks' test victories. But we place a crucial restriction on the valuation $V(i)$ of any nominal i: this must be a *singleton* subset of T. So nominals in effect are names for time in T.

Given a model $\mathfrak{M} = (T, \prec, V)$ we define truth at a time as follows:

$$\begin{array}{lll}
\mathfrak{M}, t \models a & \text{iff} & a \text{ is atomic and } t \in V(a) \\
\mathfrak{M}, t \models \neg\varphi & \text{iff} & \mathfrak{M}, t \not\models \varphi \\
\mathfrak{M}, t \models \varphi \wedge \psi & \text{iff} & \mathfrak{M}, t \models \varphi \text{ and } \mathfrak{M}, t \models \psi \\
\mathfrak{M}, t \models P\varphi & \text{iff} & \text{for some } t',\ t' \prec t \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models F\varphi & \text{iff} & \text{for some } t',\ t \prec t' \text{ and } \mathfrak{M}, t' \models \varphi \\
\mathfrak{M}, t \models \Diamond\varphi & \text{iff} & \text{for some } t',\ \text{we have } \mathfrak{M}, t' \models \varphi.
\end{array}$$

[4] Arthur Prior was the inventor of hybrid logic, a fact which is surprisingly little known given the central role they play in his work on temporal logic; this curious state of affairs is discussed in detail in (Blackburn; 2006). In the present paper we have (by and large) adopted contemporary hybrid logical notation and terminology; for example, Prior would have spoken of world-variables rather than nominals. But to make the comparison with "'Now'" more transparent we have followed Prior and used $\Diamond$ and $\Box$ for the universal modalities.

[5] Prior is insistent here: "For the present, however, let us simply consider the system $\mathrm{K_t}$ which is in a sense 'minimal'. It is well to confine ourselves to this because I want to show that it is awkward to introduce into tense-logic an operator with the properties of the idiomatic 'now', but if the tense-logic into which I introduce this operator is richer than $\mathrm{K_t}$ it is too easy to suggest that the trouble arises from my having made rash assumptions about time in the first place." (Prior; 1968/2003, p. 178).

A word on the role played by $\Diamond$ and $\Box$. Note that $\Diamond\varphi$ scans the entire model looking for a time where φ is true, while its dual form $\Box\varphi$ claims that φ is true at all times. Consider the following two schemas:

$$\Diamond(i \wedge \varphi) \qquad \Box(i \to \varphi).$$

These are equivalent. The first says: there is a point where i is true and φ is true there too. The second says: at every time where i is true, φ is true too. Prior used $\Box$ and $\Diamond$ primarily to express this, as this allowed him to capture the effect of UT calculus formulas of the form $Ti(\varphi)$ within tense logic.[6]

But where is 'now'? Easy! Just add a brand new nominal n to the language (or re-christen one of the old ones if you prefer). Then, given any model $\mathfrak{M} = (T, \prec, V)$ pick some $t_0 \in T$ and regard this designated time as the 'now' of the model. Insist that in any model, n must be true at t_0 (and nowhere else). It is immediately clear that $\Box(n \to \varphi)$ and $\Diamond(n \wedge \varphi)$ both state that φ is true now, and these are the formulas that Prior uses to define his 'now' operator J. So $J\varphi$ works like a a Kamp-style 'now' operator, but it is defined in a one-dimensional semantics and makes direct use of Casteñeda's insight: it rides on the coat-tails of a uniquely true proposition.

4 Why Prior Was Right

Let us say that a formula is *logically valid* iff it is true at all points in all models, and *contextually valid* iff it is true in all models at the designated points t_0. Clearly all logically valid formulas are contextually valid, but there are contextually valid formulas that are not logically valid: n and $\varphi \leftrightarrow J\varphi$ are two easy examples. A proof system is *logically complete* iff it generates all logical validities and *contextually complete* iff it generates all contextually valid formulas. Recall Prior's remark: "RL (modified), L1–5, A1.1–A5 with the definition of J, will I think yield all the theorems in J that we we want". He is right, and to prove this we shall split it into two claims.

Logical Completeness. Take Prior's axioms and rules but *without* A3, which is just n, the simplest axiom of all. This system is logically complete. Why? First, these axioms and rules are closely related to complete proof systems for hybrid logic with $\Diamond$, and it easy to check their adequacy; see, for example, the systems in (Gargov and Goranko; 1993). Second, it is straightforward to check that interpreting the special n nominal only on the designated point t_0 affects nothing.[7] Technical details for a related system can be found in (Blackburn and Jørgensen; 2012), but the argument is straightforward: as far as *logical* validity is concerned, n is interchangeable with any other nominal. This result lends supports to Casteñeda's conceptual analysis.

Contextual Completeness. So we have a logically complete system. Add to it A3, that is n, and we have contextual completeness. Why? First a simple semantic fact which we leave the reader to check: any formula φ is contextually valid iff $\Box(n \to \varphi)$ is

[6] These operators are nowadays often written E (there exists some time) and A (at all times) and are usually called universal modalities. They have played an important role in the development of hybrid logic (see Gargov and Goranko (1993) and Blackburn and Seligman (1995)) and are important in their own right (see Goranko and Passy (1992)).

[7] One remark. RL is the rule that from $\vdash \varphi$ we can conclude $\vdash \Box\varphi$. The modification is that this can never be applied to a formula containing occurrences of n or J. This restriction does *not* affect logical completeness. If φ is a logical validity containing occurrences of n or J, then choose a nominal k not occurring in φ and replace all occurrences of n by k. This new formula, $\varphi[n \leftarrow k]$, is also logically valid and hence provable. But then we prove φ in one more step by substituting n for k.

logically valid. But this means that we could prove any contextual validity φ if we could first prove $\Box(n \to \varphi)$ and then "peel away" the $\Box(n \to \quad)$ to reveal φ. And we *can* do this: the logic of $\Box$ is **S5**, hence from $\vdash \Box(n \to \varphi)$ we can prove $\vdash n \to \varphi$ and then, using A3 and Modus Ponens, we have φ. This establishes contextual completeness. Checking soundness takes more work; here the restrictions on RL come into play (recall the previous footnote). We leave further discussion for the full version of the paper.

5 Conclusion

This abstract has not covered all that is of interest in "'Now'". To mention some: Prior's axiomatization of the tense logic enriched with $\Box$ and J is of independent interest, as is Meredith's Property Calculus axiomatization (given on page 188). Nor have we discussed the links between Prior's work and contemporary hybrid analyses of temporal indexicals such as *yesterday*, *today* and *tomorrow*, which we believe add further weight to Prior's conviction that one-dimensional solutions can (and should) be found. Finally, we have said very little about Casteñeda's work and have not made use of the Prior-Kamp correspondence in the Prior Archive.

But we leave such matters to the full version of the paper. Here we shall simply remark on Prior's remarkable logical insight. Even with all the powerful tools of contemporary modal and tense logic at our disposal, working on the logic of indexicality is difficult. The ease with which Prior navigates between UT calculus, hybrid logic, and tense logic, seldom stumbling and never falling, is both impressive and humbling.

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