Things That Might Not Have Been

Michael Nelson
University of California at Riverside
mnelson@ucr.edu

Quantified Modal Logic (QML), to echo Arthur Prior, is haunted by the myth of necessary existence. Features of standard Quantified Logic (SQL) and Propositional Modal Logic (PML), compelling when taken individually, combine to generate the conclusion that, necessarily, absolutely everything necessarily exists. $\exists y(y=a)$ is true in every standard interpretation, as there are no empty domains and every individual constant has a value from that domain. So, given completeness, $\exists y(y=a)$ is a theorem of SQL. Even the weakest standard PML includes a Rule of Necessitation (RN), the principle that the necessitation of any theorem is itself a theorem. So, if QML is the combination of SQL and PML, we get $\Box\exists y(y=a)$ as a theorem of QML. As the same line of reasoning applies to any individual constant, $\forall x\Box\exists y(y=x)$ is a theorem of QML and so, by RN, so too is $\Box\forall x\Box\exists y(y=x)$ --- call this last sentence *NNE* --- which says that necessarily everything necessarily exists (assuming we regiment '*x* exists' as $\exists y(y=x)$).

This is problematic because ordinary objects, from people, paintings and chairs to trees and cloud formations, owe their existence to contingent happenings. Had my parents not met and had children, I would not have been; had Caravaggio not put paint to canvas, *St. Jerome* would not have been. These objects, then, do not exist as a matter of necessity. And other objects could have existed as well. Had my parents had more children than they actually had, there would have been something distinct from everything that there actually is (as nothing that there actually is could have been my parent's children except for me and my siblings); and Caravaggio could have painted more paintings than he actually painted. So, there could have been other objects than those that there actually are. What there is, then, is thoroughly contingent. These intuitions suggest that NNE is false.

There are two broad ways to deal with this problem. The first is to accept that $\Box\forall x\Box\exists y(y=x)$ is a theorem of a QML adequate for modal discourse about ordinary objects and present an alternative account of the intuitions that what exists is contingent. The second is to develop a logic in which $\Box\forall x\Box\exists y(y=x)$ is not a theorem. I defend a version of the second strategy. I follow Arthur Prior in clinging to a classical theory of quantification and faulting RN. However, I offer an importantly different philosophical understanding of its failure. While Prior distinguished between a sentence's being not possibly false and its being necessarily true, where a sentence is the first but not the second when it is not statable in every possible world, I distinguish two notions of truth with respect to a world which allows us to say that sentences like 'I do not exist' express propositions that are *false* (and not merely "unstatable") at possible worlds where I do not exist. Let *a* have o as value and let w be a possible world where o does not exist. Then both $\exists y(y=a)$ and $Fa\rightarrow\exists y(y=a)$ are unstatable with respect to w and so both have the same modal status. But there is intuitively an important difference between the two: The first is possibly false while the second isn't. Prior's account fails to account for that difference, while the account I offer does. In what follows I present a proof theory adequate for a varying domain semantics in which $\Box\forall x\Box\exists y(y=x)$ is not a theorem.

Return to the first strategy identified above for dealing with our problem: Count NNE as logically true and accept a fixed domain semantics for quantified modal discourse. Suppose that all instances of the characteristic T axiom ($\Box\phi\rightarrow\phi$) and S5 axiom ($\Diamond\phi\rightarrow\Box\Diamond\phi$) are valid, in which case, for every interpretation I, all worlds w and w' in W_I are such that w is accessible from w', allowing us to ignore the accessibility relation altogether. Then a fixed domain semantics is one in which, for any interpretation I, there is a set of individuals D that is the common domain all worlds in W_I. NNE is valid given this condition on interpretations, so we avoid the need to find fault with derivations of it. A downside is that we must explain our intuitions that existence is a contingent property. In a number of works, most recently (Williamson 2013), Timothy Williamson, following (Linsky and Zalta 1994), argues that ordinary objects are necessary existents but only contingently concrete. A possibility in which I am nonconcrete seems to ordinary intuition like a possibility in which I do not exist. Ordinary intuition systematically conflates, then, my possible nonconcreteness with my nonbeing, mistaking (or misdescribing) possibilities in which I am nonconcrete with possibilities in which I do not exist.

All versions of the second *contingentist* strategy complicate their logic in order to avoid counting NNE as a theorem of a QML adequate for modal discourse about ordinary objects. The model theory for doing this is fairly straightforward, being a varying domain semantics. The following is a simple interpretation in which $\Box\forall x\Box\exists y(y=x)$ is false. (Accessibility relations are suppressed because we again assume S5+T as a frame condition.)

Interpretation I_1: $W_{I1}=\{w^*, w_1\}$; $\{D_{w^*}=\{1,2\}, D_{w1}=\{2,3\}\}$.

1 and 3 are contingent existents, while 2 is a necessary existent. We can call 1 an *absentee*, as 1 actually exists but might not have, and 3 an *alien*, as 2 does not exist but might have. NNE is false in I_1 because $\forall x\Box\exists y(y=x)$ is false at both w^* and w_1.

A contingentist model theory is fairly straightforward. What is less clear, however, is, first, how to make metaphysical sense of that model theory (especially aliens like 3 --- do they reside in some nonactual realm? or are they not really part of the model theory at all?) and, second, what a proof theory sound and complete with respect to that model theory looks like. My purpose here is to address these issues.

We can distinguish two versions of contingentism. The first complicates the quantificational component, abandoning SQL in favor of a free logic, and the second the modal component, rejecting RN. We begin with the first. A model theory for free logic allows empty domains and empty individual constants, so $\exists y(y=a)$ is not valid in FL. Consider a natural deduction proof theory for SQL with a pair of introduction and elimination rules for the quantifier $\exists$, where the introduction rule allows the transition from any instance of $\Phi\alpha$ to a corresponding instance of $\exists x\Phi x$, where at least one occurrence of α is replaced by x and all else remains the same. An introduction rule in FL allows a transition from any instance of $\Phi\alpha$ to a corresponding instance of $\exists x(x=\alpha)\rightarrow\exists x\Phi x$. This blocks the derivation of NNE described above. The second version of contingentism is based on a rejection of RN, where there are theorems of QML whose necessitations are not theorems; in particular, $\exists y(y=a)$ is a theorem while $\Box\exists y(y=a)$ is not.

Prior developed an early version of the second version of contingentism, arguing that there are logical truths that are not true in every possible world because they are unstatable in some. For example, where a names a contingent existent, $Fa\vee\neg Fa$ is logically true, but is not a necessary truth, as it is not true in worlds where the designation of a does not exist. On Prior's view, $\exists x(x=a)$ is similarly a logical truth that it is not true in every possible world and hence $\Box\exists y(y=a)$ is not logically true. RN is false thus false and the derivation of NNE sketched above fails.

Prior was correct to fault RN. But I argue that his account is inadequate, as he did not recognize genuinely *contingent* logical truths --- that is, theorems of a sound proof theory that are possibly false, which is the status we want to ascribe to $\exists x(x=a)$. We don't merely want to say that that sentence is not true at every world but that it is in addition *false* at worlds where the value of a does not exist. Prior's account of the failure of RN does not allow that. On Prior's view, the sense in which $\Box\exists x(x=a)$ is not true is equally true of $\Box(Fa\vee\neg Fa)$, as $Fa\vee\neg Fa$ is also unstatable in worlds where the value of a does not exist. But we intuit that the first embedded sentence is false at some possibilities while the second is false at none.

A more adequate account of RN's failure arises when we, followingh (Adams 1981) and (Fine 1985), distinguish two notions of truth with respect to a nonactual possible world. ϕ is true *in* a nonactual world w just in case, were w actual, then ϕ would be true. ϕ is true *at* a nonactual world w, on the other hand, when, from the perspective of the actual world and using all of its resources, ϕ correctly characterizes how things would have been. While the proposition expressed by 'I do not exist' is not true in any possible world, as it is "statable" only when I exist, it is true at possible worlds where I do not exist. This is an improvement over Prior's notion of unstatability. I depart from Adams, however, in his account of truth at a world for modal propositions, which plays an important role in his arguments that S4 and S5 are incorrect for modal discourse. On my view, both S4 and S5 are valid and part of a modal logic adequate for our discourse about contingent existents.

I develop a natural deduction system with a standard, classical quantificational base, restricted rule of necessitation, and rules governing the actuality operator $\mathcal{A}$ (which I discuss below) that is sound and complete with respect to a classical varying domain model theory. The propositional and quantificational rules are familiar and I here only present the less familiar rules, starting with the $\Box$ rules. (T and S5 are added as inference rules.)

$\Box$I

i.	$\Box$	
j.	ϕ	*justification*
k.	$\Box\phi$	$\Box$I:i,j

$\Box$E

i.	$\Box\phi$	*justification*
j.	$\Box$	
k.	ϕ	$\Box$E:i

These rules are based on the natural deduction system presented in (Garson 2006). While typical subproofs (associated, for example, with $\rightarrow$I and $\neg$I) mark environments tainted by a temporary premise, the box-subproof marks an environment in which "only necessary truths occur." We ensure this proof-theoretically by saying that a line above the start of the box-subproof can be cited in the justification of a line inside that box-subproof only by $\Box$E. and only if it there are no further box-subproofs between. The classical quantificational rules, valid outside box-subproofs, are banned inside box-subproofs and instead only the following rules are allowed. (This corresponds to restricting RN in standard modal systems.)

$\exists E_{\Box}$

i.	$\Box$	
j.	$\exists x\Phi x$	*justification*
k.	$\boxed{\alpha}\ \exists x(x=\alpha)\rightarrow\Phi\alpha$	[temp hyp]
	:	:
l.	ϕ	*justification*
m.	ϕ	$\exists E_{\Box}$:j,k,l

$\exists I_{\Box}$

i.	$\Box$	
j.	$\Phi\alpha$	*justification*
k.	$\exists x(x=\alpha)$	*justification*
l.	$\exists x\Phi x$	$\exists I_{\Box}$:j,k

These two rules correspond to FL quantifier rules. However, they do not form the quantificational basis of the quantificational system but only the restricted quantifier rules used inside a box-subproof. Classical quantifier rules preserve truth while the free quantifier rules preserve necessary truth.

Call the above system CC-QML, for Classical Contingentist QML. The pay off is that the following derivation, valid if the classical quantifier rules are allowed to operate inside the box-subproof environment, fails. (I use the derivative universal quantifier rules for simplicity.)

0.		
1.	$\Box$	
$\boxed{2}$.	a	
3.	$\Box$	
4.	$a=a$	=I
**5.	$\exists y(y=a)$	$\exists$I:4*****
6.	$\Box\exists y(y=a)$	$\Box$I:3,5
**7.	$\forall x\Box\exists y(y=x)$	$\forall$I:2,6*****
8.	$\Box\forall x\Box\exists y(y=x)$	$\Box$I: 1,7

Lines 5 and 7 are illegitimate, as the classical quantifier rules cannot be used inside a box-subproof. Note that we cannot simply substitute the $\Box$-quantifier rules, as follows.

0.		
1.	$\Box$	
$\boxed{2}$.	$a \quad \exists y(y=a)$	hyp
3.	$\Box$	
4.	$a=a$	=I
**5.	$\exists y(y=a)$	$\exists I_{\Box}$:2,4*****
6.	$\Box\exists y(y=a)$	$\Box$I:3,5
7.	$\forall x\Box\exists y(y=x)$	$\forall I_{\Box}$:2,6
8.	$\Box\forall x\Box\exists y(y=x)$	$\Box$I: 1,7

Line 5 is illegitimate because it involves using $\exists I_{\Box}$ across a box-subproof on line 3. Derivations of NNE available in standard systems are thus blocked in CC-QML.

This system's existence shows that SQL can be modalized without turning existence into a necessary property. CC-QML is superior to Prior's system, which has some of the same features, as it does not require distinguishing strong and weak necessity and it counts $\exists y(y=a)$ not merely as not true in every world but as positively false at some worlds, keeping the interdefinability of $\Box$ and $\Diamond$ (albeit in terms of truth at worlds).

The full system includes the operator $\mathcal{A}$. $\mathcal{A}\phi$ is true in an interpretation I at a world w just in case ϕ is true at the distinguished world w* of I. The following are inference rules for the logic of $\mathcal{A}$ (based on (Zalta 1988)).

$\mathcal{A}$I

i.	ϕ	*justification*
j.	$\mathcal{A}\phi$	$\mathcal{A}$I:i

$\mathcal{A}$E

i.	$\mathcal{A}\phi$	*justification*
j.	ϕ	$\mathcal{A}$E:i

$\Box\mathcal{A}$

i.	$\mathcal{A}\phi$	*justification*
j.	$\Box\mathcal{A}\phi$	$\Box\mathcal{A}$:i

An actuality operator is needed to fully express the possibility of what I above called *aliens*, as follows: $\Diamond\exists x\neg\mathcal{A}\exists y(y{=}x)$. However, we must take care in the interaction of $\Box$, $\exists$, and $\mathcal{A}$.

There are contingent instances of $\mathcal{A}\phi\leftrightarrow\phi$, in which case we must restrict the use of $\mathcal{A}$I and $\mathcal{A}$E in box-subproof environments, much as we did with the classical quantifier rules above. But we want to count $\Box Fa\rightarrow\Box\mathcal{A}Fa$ and $\Box\forall x\Box\exists y(y{=}x)\rightarrow\Box\forall x\mathcal{A}\exists y(y{=}x)$ as theorems, as they are both logically true given our model theory. More troubling, $\forall x\Box\exists y(y{=}x)\rightarrow\Box\forall x\mathcal{A}\exists y(y{=}x)$ *should not* be provable, as it has a countermodel, as follows.

Interpretation I_2: $W_{I2}=\{w^*, w_1\}$; $\{D_{w^*}=\{1\}, D_{w1}=\{1,2\}\}$.

I know of no contingentist proof theory in the literature that delivers these results. But we get them by banning $\mathcal{A}$I and $\mathcal{A}$E from box-subproof environments and adding the following rule to govern the introduction of $\mathcal{A}$ sentences inside box-subproofs.

$\mathcal{A}$I$_\Box$

i.	$\Box$	
j.	$\Box\phi$	*justification*
k.	$\mathcal{A}\phi$	$\mathcal{A}$I$_\Box$:j

The result, call it *A-CC-QML*, is a logic in which $\Box\forall x\Box\exists y(y{=}x)\rightarrow\Box\forall x\mathcal{A}\exists y(y{=}x)$ is provable and $\forall x\Box\exists y(y{=}x)\rightarrow\Box\forall x\mathcal{A}\exists y(y{=}x)$ is not, just as we desire.

A-CC-QML is superior to Prior's own offerings, as it does not distinguish between not being possibly false because possibly unstatable and not being possibly false because genuinely necessary, allowing us to say, for example, that there are instances of $\exists x(x{=}\alpha)$ that are false at nonactual worlds. A-CC-QML is the most promising QML for contingent beings.

Bibliography

Adams, R. M., 1981, "Actualism and Thisness," *Synthese*, 49: 3–41.

Fine, K., 1985, "Plantinga on the Reduction of Possibilist Discourse," in J. Tomberlin and P. van Inwagen (eds.), *Profiles: Alvin Plantinga*, Dordrecht: D. Reidel, 145–186.

Garson, J., 2006, *Modal Logic for Philosophers*, Cambridge University Press.

Linsky, B., and Zalta, E., 1994, "In Defense of the Simplest Quantified Modal Logic," *Philosophical Perspectives* 8: 431–458.

Prior, A., 1956, "Modality and Quantification in S5," *Journal of Symbolic Logic*, 21: 60–2.

Prior, A., 1957, *Time and Modality*, Oxford: Clarendon.

Prior, A., and Fine, K., 1977, *Worlds, Times and Selves*, Amherst: University of Masssachusetts Press.

Williamson, T., 2013, *Modal Logic as Metaphysics*, Oxford: Oxford University Press.

Zalta, E., 1988, "Logical and Analytic Truths That Are Not Necessary," *Journal of Philosophy*, 85: 57–74.