Prior and temporal sequences for natural language

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In a widely quoted defense of logics of discrete time, Arthur Prior writes

> The usefulness of systems of this sort does not depend on any serious metaphysical assumption that time is discrete; they are applicable in limited fields of discourse in which we are concerned with what happens in a sequence of discrete states, e.g. in the workings of a digital computer

(Prior 1967, page 67). Forming sequences based on computational steps has been remarkably fruitful in computer science (e.g., Emerson 1995). In linguistic semantics, however, there is no obvious analog to a computational step, and tense logics, with or without the assumption of discrete time, have arguably met less success (since their adoption in Montague 1973). Moving away from discrete time, formally inclined linguists have (from Bennett and Partee 1972 on) embraced intervals that (as in the real line) are arbitrarily divisible (e.g., Dowty 1979, Kamp and Reyle 1993, van Lambalgen and Hamm 2004, Pratt-Hartmann 2005, Klein 2009). And following Reichenbach 1947 and Davidson 1967, notions of reference and event have taken center stage — so much so that "tense logic has fallen into disuse in natural language semantics" (Blackburn 2006, page 342). Focusing on the issue of discrete time, the present paper applies Prior's statement above to shed light on the widening distance between Priorean tense logic and temporal semantics in linguistics. Very briefly, "limited fields of discourse" are linked to finite sets of temporal propositions, and "a sequence of discrete states" associated with a finite automaton is induced from a choice of such a finite set. While no single choice can capture the open-endedness of ordinary language, certain choices suffice for certain purposes, yielding a serviceable notion of discrete time. Variations in these choices can lead to non-discrete time (including the real line).

Past, present and future — plus the progressive

Fix a linear order $\prec$ on a set T of instants. An instant $t \in T$ splits T into 3 disjoint subsets, $\{t\}$, past(t) and future(t) where

$$\begin{aligned} \text{past}(t) &:= \{t' \in T \mid t' \prec t\} \\ \text{future}(t) &:= \{t' \in T \mid t \prec t'\} \ . \end{aligned}$$

Assuming t is neither $\prec$-least nor $\prec$-greatest, the sets past(t) and future(t) are non-empty, and we get the chain past(t) $\prec \{t\} \prec$ future(t), where $\prec$ is lifted to intervals $I, I' \subseteq T$ via universal quantification for *whole* precedence

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\ t \prec t'.$$

Focusing on the sets past(t) and future(t), Galton 1987 defines a *formal occurrence* to be a pair (B, A) of intervals B and A such that

$$B \prec A, \quad B \prec \overline{B} \ \text{ and } \ \overline{A} \prec A$$

where the *complement* $\overline{C}$ *of* C is $\{t \in T \mid t \notin C\}$. The intuition is that the "*b*efore" set B is past(t), while the "*a*fter" set A is future(t), except that t (or better yet, $\overline{B \cup A}$) is allowed to stretch into an interval or vanish altogether into the empty set $\emptyset$. An *event radical* e is then interpreted as a set $[\![e]\!]$ of formal occurrences serving as an input/output relation between intervals

$$B[\![e]\!]A \iff e \text{ outputs } A \text{ on input } B, \text{ taking up time } \overline{B \cup A}$$

with the progessive Prog(e) of e holding at instants in $\overline{B \cup A}$ for $B[\![e]\!]A$.

(G) $$t \models \mathrm{Prog}(e) \iff (\exists B \prec \{t\})(\exists A \succ \{t\})\ B[\![e]\!]A$$

(G) is similar to an earlier account (N) of the progressive from Nishimura 1980, under which some sentences are evaluated at instants (or moments) t and others (Galton's event radicals) at intervals (t, t') with $t \prec t'$.

(N) $$t \models \mathrm{ING}(e) \iff (\exists x \prec t)(\exists y \succ t)\ (x, y) \models e$$

(N) is, in turn, a modification of a well-known proposal (S) by Dana Scott.

(S) $$t \models \mathrm{PROG}(e) \iff (\exists x \prec t)(\exists y \succ t)(\forall t' \in (x, y))\ t' \models e$$

Over the real line, an interval (x, y) around t includes instants in the past and future of t so that under (S), we have

(s) whenever $t \models \mathrm{PROG}(e)$, $(\exists t' \prec t)\ t' \models e$ and $(\exists t'' \succ t)\ t'' \models e$.

The spillover (s) reflects the "ongoing" character of imperfectives (including progressives), but is lost in (G), defeating the point of distinguishing instants from formal occurrences to capture the contrasts (1) between imperfectives and perfectives (e.g., Comrie 1976).

(1) a. imperfective: ongoing, open-ended, viewed from inside

b. perfective: completed, closed, viewed from outside

An alternative to (G) that is arguably more faithful to (1) defines an interval I to be *inside* another interval I' that stretches to the left and right of I

$$I \sqsubset I' \iff (\exists x \in I')\ \{x\} \prec I \text{ and } (\exists y \in I')\ I \prec \{y\}.$$

We can then put the distinction between imperfectives and perfectives with event time E down to a viewpoint, analyzed as an interval R, with perfectives inside R, (2b), and R inside imperfectives, (2a).

(2) a. imperfective: R $\sqsubset$ E   b. perfective: E $\sqsubset$ R

The contrast in (2) can be pictured as in (3), with an imperfective E segmented into three boxes, (3a), the middle of which contains R, and the perfective E left whole inside the middle box in (3b).

(3) a. E segmented: $\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{R}}\boxed{\mathrm{E}_\circ}$ b. E whole: $\boxed{\mathrm{R}_\circ}\boxed{\mathrm{E},\mathrm{R}_\circ}\boxed{\mathrm{R}_\circ}$

The strings of boxes in (3) are examples of the sequences mentioned by Prior above, which we interpret model-theoretically in the next section, treating E and R as temporal propositions, not unlike Areces and Blackburn 2005, except that they are evaluated at an interval (which may exceed an instant), a snapshot of which is given by a box, arranged one after another, as in a comic strip (Fernando 2013).

Segmentations and strings

Fix a set Φ of temporal propositions, or *fluents*, including E and R, and for every $\varphi \in \Phi$, the *φ-segment*, $\varphi_\circ$, satisfied by intervals I according to (4).

(4) a. $I \models \mathrm{I} \iff I = \mathrm{I} \quad$ for I $\in \{\mathrm{E}, \mathrm{R}\}$

b. $I \models \varphi_\circ \iff (\exists J \supseteq I)\ J \models \varphi$

(4a) treats E and R as names for themselves, while under (4b), φ-segments hold precisely at subintervals of φ-intervals. We extend satisfaction $\models$ to strings such as those in (3a) and (3b) by segmenting an interval I as follows. A *segmentation of I* is a finite sequence $\mathbb{I} = I_1 \cdots I_n$ of subintervals of I that partition I and are in $\prec$-order — i.e.,

$$I = \bigcup_{i=1}^{n} I_i \ \text{ and } \ I_i \prec I_{i+1} \text{ for } 1 \leq i < n.$$

A formal occurrence is just a segmentation of the entire set T of instants into 2 or 3 intervals. For brevity, we refer to a segmentation $I_1 \cdots I_n$ of $\bigcup_{i=1}^{n} I_i$ as a *seg*. Given a string $\alpha_1 \cdots \alpha_n$ of sets α_i of fluents, and a seg $I_1 \cdots I_n$ of the same length n, we define $\alpha_1 \cdots \alpha_n$ to *hold at* $I_1 \cdots I_n$, and write $I_1 \cdots I_n \models \alpha_1 \cdots \alpha_n$, if $I_i \models \varphi$ for each i from 1 to n and each $\varphi \in \alpha_i$, (5).

(5) $I_1 \cdots I_n \models \alpha_1 \cdots \alpha_n \iff$ for $1 \leq i \leq n$ and all $\varphi \in \alpha_i$, $I_i \models \varphi$

Under these conventions, the string $\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{R}}\boxed{\mathrm{E}_\circ}$ in (3a) holds at a seg $I_1 I_2 I_3$ precisely if $I_1 \cup I_2 \cup I_3 \subseteq \mathrm{E}$ and $I_2 = \mathrm{R}$. As the length n of a seg may exceed 3, we are not limited to the perfective event radicals in Galton 1987 or to the fluents R and $\mathrm{E}_\circ$.

A commonly held view (shared by the avowedly Davidsonian Taylor 1977 and Montagovian Dowy 1979) is that a fluent φ representing a state holds at an interval I precisely if if holds at every instant in I — i.e., φ is pointwise in the sense defined in (6).

(6) φ is *pointwise* if for every interval I, $I \models \varphi \iff (\forall t \in I)\ \{t\} \models \varphi$

We can, under suitable assumptions, determine which subintervals of an interval satisfy a pointwise fluent φ from a segmentation of the interval. A seg $I_1 \cdots I_n$ is *φ-fine* if a subinterval I of $\bigcup_{i=1}^{n} I_i$ satisfies φ exactly if I is covered by the seg components I_i satisfying φ

$$I \models \varphi \iff I \subseteq \bigcup \{I_i \mid 1 \leq i \leq n \text{ and } I_i \models \varphi\}.$$

Observe that if a seg is φ-fine, then so is any seg that induces a finer partition, provided φ is pointwise. Returning now to the paragraph above from Prior 1967, we identify a limited field of discourse with a finite set X of pointwise fluents, and call a seg *X-fine* if it is φ-fine for every $\varphi \in X$. Clearly, if a seg is X-fine then it is X'-fine for all $X' \subseteq X$. As we add fluents to our field X of discourse, X-fine segmentations become finer. To describe the coarsest segmentation of I that is X-fine (if it exists), some definitions are in order. The *X-diagram of* a seg $I_1 \cdots I_n$ is the string $\alpha_1 \cdots \alpha_n$ of subsets

$$\alpha_i \ = \ \{\varphi \in X \mid I_i \models \varphi\} \qquad (1 \leq i \leq n)$$

of X encoding satisfaction. A string s over the alphabet $Pow(X)$ of subsets of X is said to *X-represent* I if s is the X-diagram of some X-fine segmentation of I. We reduce all repeating blocks $\alpha\alpha^n$ in s to α for its *block compression* $\mathit{bc}(s)$

$$\mathit{bc}(s) \;=\; \begin{cases} \mathit{bc}(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha\ \mathit{bc}(\beta s') & \text{if } s = \alpha\beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise.} \end{cases}$$

For example, $\mathit{bc}(\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}) = \boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}$. In general, $\mathit{bc}(\mathit{bc}(s)) = \mathit{bc}(s)$, and $\mathit{bc}(s)$ is stutter-less in that if $\mathit{bc}(s) = \alpha_1 \cdots \alpha_n$ then $\alpha_i \neq \alpha_{i+1}$ for $1 \leq i < n$.

Lemma 1. *For any interval I, set X of pointwise fluents, and strings s and s' in $Pow(X)^*$ that X-represent I, $\mathit{bc}(s) = \mathit{bc}(s')$ and $\mathit{bc}(s)$ X-represents I.*

Lemma 1 tell us that if an interval I has an X-fine segmentation, then it has a coarsest (and shortest) X-fine segmentation. But when is there an X-fine segmentation of I? Obviously, fluents in X had better not alternate between true and false within I indefinitely. More precisely, let us call I *φ-alternation bounded* (a.b.) if the boundary of the set $\{t \in I \mid \{t\} \models \varphi\}$ is finite.[1]

Lemma 2. *For any interval I and pointwise fluent φ, there is a φ-fine segmentation of I iff I is φ-a.b.*

Next, we approximate a set Φ of pointwise fluents by the set $Fin(\Phi)$ of finite subsets of Φ (such subsets corresponding to "limited fields of discourse"). For every $X \in Fin(\Phi)$, we define the function $\mathit{bc}_X : Pow(\Phi)^* \to Pow(X)^*$ mapping a string $s \in Pow(\Phi)^*$ to the block compression of the componentwise intersection of $s = \alpha_1 \cdots \alpha_n$ with X

$$\mathit{bc}_X(\alpha_1 \cdots \alpha_n) \;=\; \mathit{bc}((\alpha_1 \cap X) \cdots (\alpha_n \cap X)).$$

For example, if $X = \{\mathrm{E}_\circ\}$,

$$\mathit{bc}_X(\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}_\circ}\boxed{\mathrm{E}_\circ}) = \boxed{\mathrm{E}_\circ} \quad \text{and} \quad \mathit{bc}_X(\boxed{\mathrm{V}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}_\circ}\boxed{\mathrm{V}_\circ}) = \boxed{}\boxed{\mathrm{E}_\circ}\boxed{}.$$

The *inverse limit $\mathfrak{IL}(\Phi)$ of the system of functions* $\{\mathit{bc}_X \mid X \in Fin(\Phi)\}$ is the set of functions $f : Fin(\Phi) \to Pow(\Phi)^*$ such that for all $X, X' \in Fin(\Phi)$,

$$f(X) \;=\; \mathit{bc}_X(f(X')) \;\text{ whenever } X \subseteq X'.$$

A *Φ-representation of* an interval I is a function $f \in \mathfrak{IL}(\Phi)$ such that $f(X)$ X-represents I for every $X \in Fin(\Phi)$. While the partition $f(X)$ of I induced by any given $X \in Fin(\Phi)$ is finite (amounting to discrete time), we might, by expanding X along a chain $X \subset X_1 \subset X_2 \subset \cdots$ in $Fin(\Phi)$, refine $f(X)$ indefinitely.

Theorem 3. *For any interval I and set Φ of pointwise fluents, there is a Φ-representation of I iff for every $\varphi \in \Phi$, I is φ-a.b.*

[1] We assume here the order topology on T (given by unions of sets (t, t') of instants $\prec$-between t and t'), relative to which the *boundary of* a subset A of T is the closure of A minus the interior of A. Then I is φ-a.b. iff there is a largest integer n for which there exists instants $t_1 \prec t_2 \prec \cdots \prec t_n$ in I such that $\{t_i\} \models \varphi$ iff not $\{t_{i+1}\} \models \varphi$ (for $1 \leq i < n$).

By Theorem 3, the inverse limit $\mathfrak{IL}(\Phi)$ represents the whole gamut of intervals that are for every $\varphi \in \Phi$, φ-a.b., ranging over wildly different notions $\models$ of satisfaction. What structure does time have in $\mathfrak{IL}(\Phi)$? Let $\prec_\Phi$ be the binary relation on $\mathfrak{IL}(\Phi)$ given for all f and $f' \in \mathfrak{IL}(\Phi)$ by

$$f \prec_\Phi f' \iff f \neq f' \text{ and } (\forall X \in \mathit{Fin}(\Phi))\ f(X) \text{ is a prefix of } f'(X)$$

where s is a *prefix of* s' if $s' = s\hat{s}$ for some possibly empty string $\hat{s}$. Under $\prec_\Phi$, time branches insofar as $\prec_\Phi$ is transitive and for all $f \in \mathfrak{IL}(\Phi)$ and $f_1, f_2 \prec_\Phi f$,

$$f_1 \prec_\Phi f_2 \text{ or } f_2 \prec_\Phi f_1 \text{ or } f_1 = f_2.$$

The intuition is that an $f \in \mathfrak{IL}(\Phi)$ encodes the instant that is X-approximated, for each $X \in \mathit{Fin}(\Phi)$, by the last box in $f(X)$, with past given by the prefix of $f(X)$ leading to that box.

We can also equate instants with propositions, as Prior did (Øhrstrøm and Hasle 1993, page 35). In the present set-up, instants as propositions are conditioned by the choice of an $f \in \mathfrak{IL}(\Phi)$ and perhaps also a limited field of discourse $X \in \mathit{Fin}(\Phi)$. Given a proposition $\varphi \in \Phi$ and a string $s = \alpha_1 \cdots \alpha_n \in \mathit{Pow}(\Phi)^+$, call φ an *s-instant* if $\varphi \in \alpha_i$ for a unique i between 1 and n. Then for $f \in \mathfrak{IL}(\Phi)$, φ is an *f-interval* if φ is an $f(\{\varphi\})$-instant (i.e., $f(\{\varphi\}) \in \boxed{\varphi} + \boxed{}\boxed{\varphi} + \boxed{\varphi}\boxed{} + \boxed{}\boxed{\varphi}\boxed{}$). Moreover, φ is an *f-instant* if for all $\psi \in \Phi$, φ is an $f(\{\varphi, \psi\})$-instant — equivalently, φ is an f-interval and for every $X \in \mathit{Fin}(\Phi)$, φ appears in at most one position in $f(X)$. Conceiving instants this way is arguably in line with Prior's rejection of "the idea of temporal instants as something primitive and objective" (Øhrstrøm and Hasle 1993, page 33). As van Benthem puts it,

> the orthodox road in tense logic goes *from points to intervals* of time, and thence to intervals plus linguistic description of what is going on during these. But, for various philosophical and linguistical reasons, the proper order of analysis might well be the other way around. Events form the stock of our primary experience, periods are already abstract substrata underlying simultaneous events, and points are ideal limiting cases of periods. Thus, the heterodox road *from events to periods to points* deserves closer scrutiny

(van Benthem 1984, pages 4,5). We can travel along both orthodox and heterodox roads at different bounded granularities $X \in \mathit{Fin}(\Phi)$. Indeed, we can build the heterodox road on strings $\alpha_1 \cdots \alpha_n \in \mathit{Pow}(X)^+$, construed as models; we take string positions as instants for the set $T = \{1, \ldots, n\}$, with the understanding that for every interval $I \subseteq T$ (under the usual ordering $<$) and every $\varphi \in X$,

$$I \models \varphi \iff \varphi \in \bigcap_{i \in I} \alpha_i$$

for "what you see is all there is" (WYSIATI, Kahneman 2011). Moens and Steedman 1988 describe event nuclei (at the subatomic level of Parsons 1990) and episodes (at the macro-level of discourse) that we can analyze up to a bounded granularity X, representing temporal sequences as strings of subsets of X. It is not difficult to trace these strings to finite automata — in Prior's words, "the workings of a digital computer."

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