Prior and Greniewski on Aristotle’s Logical Squares

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Aristotle’s squares of opposition have fascinated logicians for over two millennia. Papers which deal with these logical queries are published every year. Arthur Prior, whose approach to this problem is analysed, discussed logical squares in several publications. However, this text deals with Prior’s unpublished paper *Aristotle on Logical Squares*, which is deposited in Bodleian Library. Prior tries here to define Greniewski’s □ operator, which Greniewski introduced in his paper *Próba “odmlodzenia” kwardatu logicznego*, but his definitions seem to be insufficient. The reasons why it was difficult for Prior to define Greniewski’s □ operator precisely will be provided in a comparison between a part of Greniewski’s paper and Prior’s attempts at defining Greniewski’s □ operator in his unpublished paper.

However, two affairs have to be stressed before the presentation of Greniewski’s and Prior’s conceptions begins. Firstly, the title of Prior’s paper as well as the title of this text contain plural ‘logical squares’. This means that not even the square of opposition, which is defined in the 7th chapter of Aristotle’s *De Interpretationes* and which is the most renowned, is examined here. Despite this square of opposition, Prior, in his attempt to define Greniewski’s □ operator, also considers squares which are introduced in the 10th and 12th chapters of *De Interpretatione* i.e. the square that comprises indefinite names and the modal square. Nonetheless, Greniewski only uses the first type of a square of opposition. Secondly, it is very probable that Prior’s unpublished paper is not based on Greniewski’s paper but is a reaction to a review of this paper that was published in the *Journal of Symbolic Logic* and which was written by Johannes Bendiek.

Greniewski

Although, Greniewski proves in his *Elementy logiky formalnej* (*Elements of Formal Logic*) that he is familiar with the history of logic, it is not his aim in *Próba "odmlodzenia" kwardatu logicznego* to reconstruct Aristotle's square of opposition. This is obvious even from the title of the Greniewski paper ("*odmlodzenia*" means "rejuvenation"). Moreover, Aristotle is not even mentioned in the paper.

Greniewski's paper begins with criticism of the concept of the logical square, which was rejected by modern logicians but which is still found in textbooks. According to him, textbooks should be a model of precision and a square of opposition is far from this ideal. This is the reason he offers his own new concept of the square of opposition, which he considered to be more precise.

The square of opposition is reduced in Greniewski's paper into the $\Box$ operator, which differs from classical logic operators like $\rightarrow$, $\wedge$, or $\vee$ in one important feature, namely that it does not hold two propositions but rather four propositions. They represent the propositions of the former square of opposition.

Greniewski's $\Box$ operator is defined:

$$\Box\begin{pmatrix} p_1\, q_1 \\ p_2\, q_2 \end{pmatrix} (p_1 \rightarrow p_2) \wedge \neg(p_1 \leftrightarrow q_2) \wedge \neg(p_2 \leftrightarrow q_1)$$

and Greniewski also defined the negation of the $\Box$ operator:

$$\neg\Box\begin{pmatrix} p_1\, q_1 \\ p_2\, q_2 \end{pmatrix} (\neg p_1 \wedge \neg p_2) \wedge (p_1 \leftrightarrow q_2) \wedge (p_2 \leftrightarrow q_1)$$

The truth values of the $\Box$ operator are these:

$p_1\, p_2$ \ $q_1\, q_2$	0 0	0 1	1 0	1 1
0 0	0	0	0	1
0 1	0	1	0	0
1 0	0	0	0	0
1 1	1	0	0	0

And these truth values are held by the negation of the □ operator:

p_1 p_2 \ q_1 q_2	1 1	1 0	0 1	0 0
1 1	1	1	1	0
1 0	1	0	1	1
0 1	1	1	1	1
0 0	0	1	1	1

Further applications of the □ operator occur in Greniewski's paper. He also demonstrates the logical laws of the square that Aristotle and his successors defined and which his operator is capable of transcribing as a contradiction, rules of contraries, subcontraries and subalterns. Nonetheless, he also adopts laws that have his operator but not Aristotle's squares as is the overturn of the sides of a square or the rules of symmetry and transitivity etc. Taking everything into account, Greniewski correctly defined the operator which can, according to him, replace the traditional square of opposition in the curriculum of Logic. He was convinced that the traditional square should be displaced from the curriculum to the history of logic.

Prior

Prior discusses logical squares in several papers and books. Some of his ideas are highly impressive, or at least surprising. Geach and Kenny point out that Prior in his *Introduction* of the book *The Doctrine of Propositions and Terms* correctly reconstructed Aristotle's concept of the existential import. Moreover, Prior's reformulations of logical squares, which he presents in his *Formal Logic,* are also very interesting. Since Prior is inspired here a great deal by Polish logicians, his reconstruction of Aristotle's squares of opposition is influenced by their logical systems. However, since Prior's unpublished text is our current subject of enquiry, we will focus primarily on the logical squares that he introduces in this paper.

Prior may have only read Bendieks's review of Greniewski's paper, while in his article he did not analyse Greniewski's criticism of traditional logic. This criticism was clearly expressed in Greniewski's paper but was not mentioned in Bendiek's review of Greniewski's paper. This could be the reason why Prior tried hard to find the form of Aristotle's squares that corresponds with Greniewski's function. This caused several difficulties as Prior endeavours to combine traditional logic with the ideas of the logician who rejects traditional logic.

There are four reconstructions of Aristotle's square of opposition in Prior's paper. Firstly, Prior replied to Greniewski's formalization of the □ operator and deduced the other laws of the square of opposition. He also discussed the simplification of the square that Greniewski postulated at the end of his article. Nevertheless, Prior's further steps were not in accordance with Greniewski's intentions. Prior used strict implication instead of Greniewski's material implication, and he extended Greniewski's definition to cover the definition of the modal square of opposition. Furthermore, Prior also discussed the square that dealt with the negation of predicates. Thirdly, Prior analysed formalization of the square of opposition in the way that it was postulated by the Polish logicians Łukasiewicz and Bocheński. The formalization follows Aristotle's ideas therefore the relations of the square of opposition did not disappear from their logical systems. Finally, Prior constructed the formalization of the square in modern predicate logic. Quantifiers and term variables are used here in order to properly describe Aristotle's square of opposition. Aristotle did not use term variables and quantifiers and Prior knew it, although according to Prior the form of Aristotle's propositions was close to this formalization.

In conclusion, there are two different intentions which led Greniewski and Prior to the formulations of their papers and two different approaches were used. Greniewski's paper is an attempt at a new approach to the square of opposition. Prior's formalisation of the □ operator, on the other hand, was dependant on logical tradition, mainly on Aristotle's own definition of logical squares. As a result, Prior was not really successful in his effort to formalise Greniewski's operator correctly and so Prior's paper remained unpublished and may also be unfinished. In addition, as far as I know Greniewski's carefully defined operator was not adopted.